

Appendices

A Equilibrium Coalition Results with Pre-Existing Taxes

Table 1 describes the results of a sensitivity analysis in which we calibrate and run counterfactual scenarios in a version of the model which includes the pre-existing tax structure described by the GTAP database. The equilibrium results for these simulations are reported in the same manner as in Tables II and IV in the main text of the paper. They indicate that including these taxes has only limited impact on the model results. In particular, USA, EUR, CHN remains the ‘best’ SPNE coalition and achieves comparable emission reductions and %EV as without taxes (see Table II).

Table 1: Coalitions by Emission Reduction and Welfare Change, 2015, w/ Pre-Existing Taxes

Homogenous Trade (σ_{DM}, σ_{MM}) = (8, 16)

	% Equivalent Variation						Global %EV	Emission Reduction
	USA	JPN	EUR	CHN	FSU	ROW		
First-Best	1.7	8.0	6.4	-3.8	-3.0	-0.1	2.2	22.7
Shapley (eur,chn)	1.2	4.4	0.7	0.7	8.2	0	1.5	14.6
JPN, EUR, CHN, FSU	1.2	3.8	2.5	0.6	11.6	0	1.5	14.6
USA, JPN, EUR, CHN, FSU	1.1	3.5	1.8	1.7	15.9	0	1.4	14.3
eur,chn,fsu**	1.1	4.1	2.3	0.5	10.2	0	1.4	14.0
JPN, EUR, CHN	1.1	3.7	2.5	0.4	6.8	0	1.4	13.8
USA, JPN, EUR, CHN	1.1	3.7	2.0	1.6	6.7	0	1.3	13.8
USA, EUR, CHN, FSU	1.0	4.0	1.7	1.5	14.3	0	1.3	13.7
USA, EUR, CHN	1.1	3.9	1.9	1.5	6.4	0	1.3	13.5
EUR, CHN**	1.1	3.9	2.4	0.3	6.5	0	1.3	13.4
USA, JPN, CHN	0.8	2.2	2.5	0.9	5.6	0	1.1	12.1
USA, JPN, CHN, FSU	0.6	2.1	2.3	0.9	8.2	0	1.1	11.8
JPN, CHN, FSU	0.7	2.2	2.0	0.2	3.9	0	0.9	10.9
USA, CHN**	0.5	2.5	1.9	0.6	4.3	0.1	0.9	10.7
USA, CHN, FSU	0.5	2.4	1.9	0.7	5.8	0.1	0.9	10.7
JPN, CHN**	0.7	2.1	1.8	0.1	4.2	0	0.9	10.5
CHN, FSU*	0.5	1.8	1.4	0.1	2.0	0	0.7	9.3
USA, JPN, EUR, FSU	0.2	0.6	-0.3	0	10.2	0	0.2	7.2
JPN, EUR, FSU	0.2	0.5	0.2	0	3.4	0	0.2	7.1
USA, EUR, FSU	0.2	0.7	-0.4	0	9.6	0	0.2	7.0
EUR, FSU*	0.2	0.6	0.2	0	2.5	0	0.2	6.8
USA, EUR	0.2	0.5	-0.2	0	0.5	0	0.1	6.5
USA, JPN, EUR	0.2	0.8	-0.3	0	0.5	0	0.1	6.5
JPN, FSU**	0.1	0.1	0.3	0	0.7	0	0.1	6.4
USA, JPN, FSU	0	-0.4	0.2	0	4.4	0	0.1	6.1
JPN, EUR**	0	0	0	0	0.2	0	0	6.0
No-Trade Nash	0	0	0	0	0	0	0	5.7
USA, JPN	0	-0.2	0	0	0	0	0	5.6
USA, FSU	-0.1	-0.1	-0.1	0	2.5	0	0	5.5

* indicates that a coalition is a SPNE.

** indicates a SPNE coalition which satisfies the weak external stability condition.

% Equivalent Variation: % change in money-metric utility from no-trade Nash.

Emission Reduction: % reduction in global emissions from BaU.

Global %EV: global equivalent variation as % change from no-trade Nash.

B Solution Method

This section provides an overview of the numerical methods which we have developed to solve our hybrid game-theory / market equilibrium model. This material is, to our knowledge, novel and of interest on in its own right, as it provides a clean and general approach to solving hybrid models.

The key challenge in computing game-theoretic equilibria is to represent the first-order conditions formulated in terms of local sensitivity of prices to strategic instruments, e.g. $d\pi_{ir}/d\omega_r^E$ and $de_{row}(\pi)/d\omega_r^E$ in (9). The implicit function theorem is the standard tool for computing the local sensitivity of the endogenous variables of a model to changes in exogenous variables. In the present context, this implies the following matrix equation:

$$\left[\frac{dz}{d\omega} \right] = - \left[\frac{dF}{dz} \right]^{-1} \left[\frac{dF}{d\omega} \right]. \quad (1)$$

The literal implementation of this system of equations creates considerable modeling overhead, as it would require explicit programming of the Jacobian matrices $\left[\frac{dF}{dz} \right]$ and $\left[\frac{dF}{d\omega} \right]$.

The local dependence of endogenous variables on exogenous variables can alternatively be approximated by solving a set of additional equilibria with small perturbations of the permit allocations. That is:

$$\frac{dz_k}{d\omega_r^E} \approx \frac{z_k(\omega + \delta^r) - z_k(\omega)}{\delta}, \quad (2)$$

where:

$$\delta^r = \begin{pmatrix} \delta_1^r \\ \vdots \\ \delta_N^r \end{pmatrix} \quad \text{with} \quad \delta_j^r = \begin{cases} \delta & r = s \\ 0 & r \neq s. \end{cases} \quad (3)$$

While numerical differentiation is typically an inferior numerical technique due to the resulting poor precision and efficiency, in the present application it offers significant savings in implementational cost.¹

At first glance it would appear that the calculation of local sensitivity of prices π_{ir} with respect to permit allocations ω_r^E would not, by itself, solve the model. The first-order conditions for the permit allocations of strategic countries are indeed *functions* of the local derivatives. The trick is to *simultaneously* solve the model and approximate the partial derivatives, $d\pi_{ir}/d\omega_r^E$. We do this by solving an $N \times n + n - 1$ equation system for a model with $n - 1$ strategic states. The central equations which characterize the economic equilibrium and its local sensitivity analysis are:

$$F(z; \omega) = 0,$$

$$F(z^r; \omega + \delta^r) = 0, \quad r \neq \text{ROW}.$$

We compute z and the adjacent perturbed solutions z^i simultaneously, together with permit allocations as endogenous variables. The permits are made endogenous through the introduction of difference approximations of the first-order conditions which characterize the Nash equilibrium permit allocation:

$$\frac{1}{p_r^c} \left[\pi_E + (\omega_r^E - e_r) \frac{\pi_E^{rr} - \pi_E}{\delta} + \Delta_r \right] - \nu_r \left(1 + \frac{e_{row}^{rr} - e_{row}}{\delta} \right) = 0 \quad , \quad \forall r \in \mathcal{C}$$

$$\frac{1}{p_r^c} [\pi_{Er} + \Delta_r] - \nu_r \left(1 + \frac{e_{row}^{rr} - e_{row}}{\delta} \right) = 0 \quad , \quad \forall r \notin \{\mathcal{C}, \text{row}\}$$

$$\Delta_r = \left(\sum_{i \neq E} (Y_{ir} - C_{irr} - I_{irr}) \frac{\pi_{ir}^{rr} - \pi_{ir}}{\delta} - \sum_{s \neq r} (C_{isr} + I_{isr}) \frac{\pi_{is}^{rr} - \pi_{is}}{\delta} \right)$$

¹Calculation of $\nabla_z F$ is sufficiently tedious to warrant the cost of conducting computational tests to determine an appropriate difference interval, δ (see Section 8.6., Gill et al (1981).)

C The Economic Impacts Model

Table 2 lists the dimensions of the economic model. The model describes a general equilibrium in geographical regions, sectors of the economy, and the primary factors each region holds. Most of the sectoral detail centers around the production of energy and energy-intensive goods, as the impact of changes in a region's emission constraint are focused here.

Benchmark data on quantities, prices, and elasticities provide the calibration point for the production and utility functions that describe the economy. The underlying data base is GTAP5 for the year 1998 which provides a consistent representation of energy markets in physical units as well as and detailed accounts of regional production, consumption, and bilateral trade flow (see Dimaranan and McDougall (2002), Rutherford and Paltsev (2000)).

Key assumptions and notation:

- Nested separable constant elasticity of substitution (CES) functions characterize the use of inputs in production. All production exhibits non-increasing returns to scale. Goods are produced with capital, labor, energy, and emission permits (KLE).
- A representative agent (RA) in each region controls these primary factors. The RA maximizes utility from consumption of a CES composite subject to a budget constraint with fixed investment demand (i.e. fixed demand for the savings good). The aggregate consumption bundle combines demands for fossil fuels, electricity and non-energy commodities. Total income of the RA consists of factor income including revenues from permit sales.
- All goods are differentiated by region of origin. Regarding imports, nested CES functions characterize the choice between imported and domestic varieties of the same good (Armington).

Table 2: Elements of the Model

Time Horizon	2000-2020
Regions	
EUR	Europe (EU15, EFTA)
JPN	Japan
USA	United States
CHN	China
FSU	Former Soviet Union
ROW	Rest of World
Sectors	
COL	Coal
CRU	Crude oil
GAS	Natural gas
OIL	Refined oil products
ELE	Electricity
EIS	Energy-intensive sectors
Y	Other economic activity
Primary Factors	
L	Labor
K	Capital
R_{ff}	Fossil fuel resources (coal, oil and natural gas)
E	CO_2 emission permits

- Labor and capital are mobile within domestic borders but cannot move between regions; natural resources are sector specific.

D Algebraic Model Description

Following Mathiesen (1985), two classes of conditions characterize the competitive equilibrium for our model: zero profit conditions and market clearance conditions. Zero profit conditions determine activity levels, while market clearance dictates the price levels. In our algebraic exposition, the notation Π_{rk}^u is used to denote the profit function of region r in sector k where u is the name assigned to the associated production activity. Differentiating the profit function with respect to input and output prices yields compensated demand and supply coefficients (Shepard's lemma), which appear subsequently in the market clearance conditions.

Tables 3 - 8 explain the notations for variables and parameters employed within our algebraic exposition. Table 4 summarizes the activity variables of vector y , whereas Table 5 summarizes the price variables of vector π . Figures 1 - 4 provide a graphical exposition of the production and final consumption structure.

D.1 Zero Profit Conditions

1. Production of goods except fossil fuels:

$$\begin{aligned} \Pi_{rk}^Y &= \pi_{rk}^Y - \sum_{j \notin FE} \theta_{jik} \pi_{ji}^A \\ &\quad - \theta_{rk}^{KLE} \left[\theta_{rk}^N (\pi_{rk}^N)^{1-\sigma_{KLE}} + (1 - \theta_{rk}^N) (w_r^{\alpha_{rk}} r_r^{1-\alpha_{rk}})^{1-\sigma_{KLE}} \right]^{1/(1-\sigma_{KLE})} \\ &= 0 \quad \forall k \notin FF \end{aligned}$$

2. Production of fossil fuels:

$$\begin{aligned}\Pi_{rk}^Y &= \pi_{rk}^Y - \left[\theta_{rk}^R q_{rk}^{1-\sigma_{Rik}} + (1 - \theta_{rk}^R) \left(\theta_{Lik}^{FF} w_r + \theta_{Kik}^{FF} r_r + \sum_j \theta_{jik}^{FF} \pi_{jr}^A \right)^{1-\sigma_{Rik}} \right]^{1/(1-\sigma_{Rik})} \\ &= 0 \quad \forall k \in FF\end{aligned}$$

3. Sector-specific energy aggregate:

$$\begin{aligned}\Pi_{rk}^N &= \pi_{rk}^N - \left\{ \theta_{rk}^{ELE} (\pi_{ELE,i}^A)^{1-\sigma_{ELE}} \right. \\ &\quad + (1 - \theta_{rk}^{ELE}) \left[\theta_{rk}^{COL} (\pi_{col,i}^N + \pi_r^E \epsilon_{rk}^{COL})^{1-\sigma_{COL}} \right. \\ &\quad \left. \left. + (1 - \theta_{rk}^{COL}) \left(\sum_{j \in LQ} \theta_{rk}^j (\pi_{ji}^A + \pi_r^E \epsilon_{rk}^j)^{1-\sigma_{LQ}} \right)^{(1-\sigma_{COL})/(1-\sigma_{LQ})} \right]^{(1-\sigma_{ELE})/(1-\sigma_{COL})} \right\}^{1/(1-\sigma_{ELE})} \\ &= 0\end{aligned}$$

4. Armington aggregate:

$$\begin{aligned}\Pi_{rk}^A &= \pi_{rk}^A - \left[\theta_{rk}^D (\pi_{rk}^Y)^{1-\sigma_{DM}} + (1 - \theta_{rk}^D) (\pi_{rk}^M)^{1-\sigma_{DM}} \right]^{1/(1-\sigma_{DM})} \\ &= 0\end{aligned}$$

5. Aggregate imports across import regions:

$$\begin{aligned}\Pi_{rk}^M &= \pi_{rk}^M - \left(\sum_s \theta_{ksi}^M (\pi_{sk}^Y + \mu_{ksi} \pi^T)^{1-\sigma_{MM}} \right)^{1/(1-\sigma_{MM})} \\ &= 0\end{aligned}$$

6. Household consumption demand:

$$\begin{aligned}\Pi_r^C &= p_r^c - \left(\theta_{Ci}^N (\pi_{Ci}^N)^{1-\sigma_C} + (1 - \theta_{Ci}^N) \left[\prod_{j \notin E} (\pi_{ji}^A)^{\theta_{ji}^C} \right]^{1-\sigma_C} \right)^{1/(1-\sigma_C)} \\ &= 0\end{aligned}$$

7. Household energy demand:

$$\begin{aligned}\Pi_{rC}^E &= \pi_{rC}^N - \prod_{j \in E} (\pi_{ji}^A + \pi_r^E \epsilon_{rC}^j)^{\theta_{ji}^N} \\ &= 0\end{aligned}$$

D.2 Market Clearance Conditions

8. Labor:

$$\omega_r^L = \sum_k Y_{rk} \frac{\partial \Pi_{rk}^Y}{\partial w_r}$$

9. Capital:

$$\omega_r^K = \sum_k Y_{rk} \frac{\partial \Pi_{rk}^Y}{\partial r_r}$$

10. Natural resources:

$$\omega_{rk}^R = Y_{rk} \frac{\partial \Pi_{rk}^Y}{\partial q_{rk}} \quad \forall k \in FF$$

11. Sectoral output:

$$Y_{rk} = A_{rk} \frac{\partial \Pi_{rk}^A}{\partial \pi_{rk}^Y} + \sum_{s \neq i} M_{sk} \frac{\partial \Pi_{sk}^M}{\partial \pi_{rk}^Y}$$

12. Sector specific energy demand:

$$N_{rk} = Y_{rk} \frac{\partial \Pi_{rk}^Y}{\partial \pi_{rk}^N}$$

13. Import supply:

$$M_{rk} = A_{rk} \frac{\partial \Pi_{rk}^A}{\partial \pi_{rk}^M}$$

14. Aggregate supply:

$$A_{rk} = \sum_j Y_{ji} \frac{\partial \Pi_{ji}^Y}{\partial \pi_{ki}^A} + C_r \frac{\partial \Pi_r^C}{\partial \pi_{ki}^A}$$

15. Household energy consumption:

$$N_{rC} = C_r \frac{\partial \Pi_r^C}{\partial \pi_{rC}^N}$$

16. Carbon emissions:

$$\omega_r^E + X_r = N_{rC} \frac{\partial \Pi_{rC}^N}{\partial \pi_r^E} + \sum_k N_{rk} \frac{\partial \Pi_{rk}^N}{\partial \pi_r^E}$$

17. International permit market:

$$\sum_{r \in \mathcal{C}} X_r = 0$$

D.3 Income balance

$$p_r^c (C_r - \bar{V}_r) = w_r \omega_r^L + r_r \omega_r^K + \sum_{j \in FF} q_{ji} \omega_{ji}^R + \pi_r^E \omega_r^E + \bar{B}_r$$

Table 3: Sets

i, j, k	Sectors and goods
r, s	Regions
EG	All energy goods: Coal, crude oil, refined oil, gas and electricity
FF	Primary fossil fuels: Coal, crude oil and gas
FE	Final energy goods: Coal, gas and refined oil
LQ	Liquid fuels: Refined oil and gas
\mathcal{C}	Emission trading coalition members

Table 4: Activity Variables

Y_{rk}	Production in region r and sector k
N_{rk}	Aggregate energy input in region r and sector k
M_{rk}	Aggregate imports of region r and sector k
A_{rk}	Armington aggregate for region r in sector k
C_r	Aggregate household consumption in region r
N_{Ci}	Aggregate household energy consumption in region r
X_r	Net exports in carbon permits in region r

Table 5: Price Variables

π_{rk}	Output price of good k produced in region r for domestic market
π_{rk}^X	Output price of good k produced in region r for export market
π_{rk}^N	Price of aggregate energy in region r and sector k
π_{rk}^M	Import price aggregate for good k imported to region r
π_{rk}^A	Price of Armington good k in region r
π^T	Price of international transport
p_r^C	Price of aggregate household consumption in region r
π_{Ci}^N	Price of aggregate household energy consumption in region r
w_r	Wage rate in region r
r_r	Price of capital services in region r
q_{rk}	Rent to natural resources in region r ($k \in FF$)
π_r^E	Carbon tax in region r

Table 6: Cost Shares

θ_{rk}^X	Share of exports in region r and sector k
θ_{jik}	Share of intermediate good j in region r and sector k ($k \notin FF$)
θ_{rk}^{KLE}	Share of KLE aggregate in region r and sector k ($k \notin FF$)
θ_{rk}^N	Share of energy in the KLE aggregate of region r and sector k ($k \notin FF$)
α_{rk}	Share of labor region r and sector k ($k \notin FF$)
θ_{rk}^R	Share of natural resources in region r of sector k ($k \in FF$)
θ_{Tik}^{FF}	Share of good j ($T = j$) or labor ($T = L$) or capital ($T = K$) in region r and sector k ($k \in FF$)
θ_{rk}^{COL}	Share of coal in fossil fuel demand by region r in sector k ($k \notin FF$)
θ_{rk}^{ELE}	Share of electricity in energy demand by region r in sector k
β_{jik}	Share of liquid fossil fuel j in energy demand by region r in sector k ($k \notin FF, j \in LQ$)
θ_{sik}^M	Share of imports of good k from region s to region r
θ_{rk}^D	Share of domestic variety in Armington good k of region r
θ_{rC}^N	Share of fossil fuel composite in aggregate household consumption in region r
θ_{rk}^C	Share of non-energy good k in non-energy household consumption demand in region r
θ_{rkC}^N	Share of fossil fuel k in household energy consumption in region r

Table 7: Endowments and Emission Coefficients

ω_r^L	Aggregate labor endowment for region r
ω_r^K	Aggregate capital endowment for region r
ω_{rk}^R	Endowment of natural resource k for region r ($k \in FF$)
\bar{B}_r	Balance of payment deficit or surplus in region r (note: $\sum_r \bar{B}_r = 0$)
\bar{V}_r	Capital investment in region r
ω_r^E	Carbon emission permit endowment for region r
ϵ_{rk}^j	Carbon emission coefficient for fossil fuel j in region r in sector k ($j \in FE$)
ϵ_{rC}^k	Carbon emission coefficient for household energy demand for fossil fuel k in region r ($k \in FE$)

Table 8: Elasticities

σ_{KLE}	Substitution between energy and value-added in production (except fossil fuels)	0.5
$\sigma_{R,i}$	Substitution between natural resources and other inputs in fossil fuel production calibrated consistently to exogenous supply elasticities μ_{FF} .	$\mu_{COA} = 1.0$ $\mu_{CRU} = 1.0$ $\mu_{GAS} = 1.0$
σ_{ELE}	Substitution between electricity and the fossil fuel aggregate in production	0.1
σ_{COL}	Substitution between coal and the liquid fossil fuel composite in production	0.5
σ_{LQ}	Substitution between liquid fossil fuels in production	2
σ_{DM}	Substitution between the import aggregate and the domestic input	8
σ_{MM}	Substitution between imports from different regions	16
σ_C	Substitution between the fossil fuel composite and the non-fossil fuel consumption aggregate in household consumption	0.5

E Production Structure

The following figures give a graphical description of the various production technologies in the model. The top level in each figure represents the output, while all subsequent levels of the tree structure describe the nesting structure of the inputs in the nested constant elasticity of substitution production functions. The substitution patterns for each nest are listed in italics at each node of the tree. *CES* denotes the general form of the function, while other labels (i.e. *Leontief* or *Cobb-Douglas (C-D)*) correspond to specific elasticity values (0 or 1 respectively). All elasticity values appear in Table 8.

Figure 1: Nesting in non-fossil fuel production

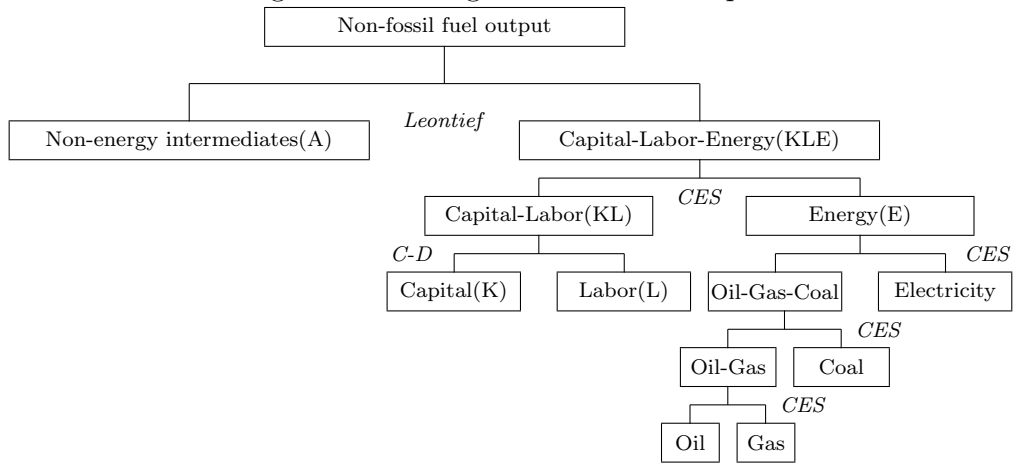


Figure 2: Nesting in fossil fuel production

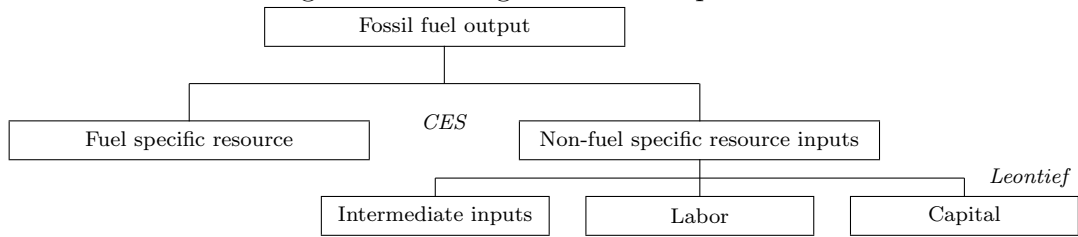


Figure 3: Nesting in household consumption

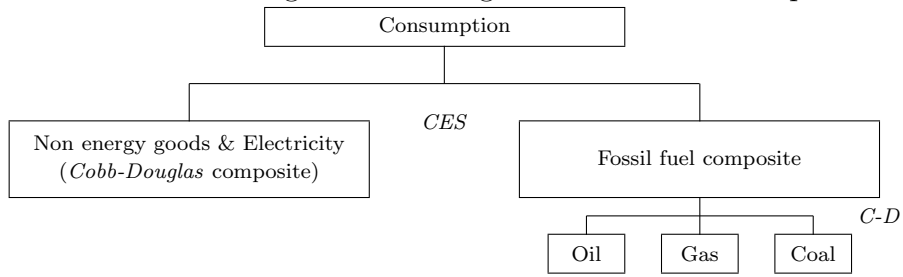
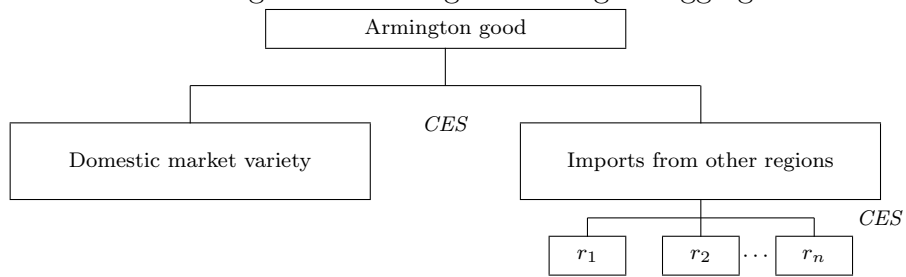


Figure 4: Nesting in Armington aggregate bundle



F Benchmark Data - Regional and Sectoral Aggregation

The model is built on a comprehensive energy-economy data set that accommodates a consistent representation of energy markets in physical units as well as detailed accounts of regional production and bilateral trade flow. The underlying data base is GTAP-EG which reconciles the GTAP economic production and trade data set for the year 1997 with OECD/IEA energy statistics for 45 regions and 22 sectors (Rutherford and Paltsev, 2000). Benchmark data determine parameters of the functional forms from a given set of benchmark quantities, prices, and elasticities. Sectors and regions of the original GTAP-EG data set are aggregated according to Tables 9 and 10 to yield the model's sectors and regions (see Table 2).

Table 9: Sectoral Aggregation

Sectors in GTAP-EG			
AGR	Agricultural products	NFM	Non-ferrous metals
CNS	Construction	NMM	Non-metallic minerals
COL	Coal	OIL	Refined oil products
CRP	Chemical industry	OME	Other machinery
CRU	Crude oil	OMF	Other manufacturing
DWE	Dwellings	OMN	Mining
ELE	Electricity and heat	PPP	Paper-pulp-print
FPR	Food products	SER	Commercial and public services
GAS	Natural gas works	T_T	Trade margins
I_S	Iron and steel industry	TRN	Transport equipment
LUM	Wood and wood-products	TWL	Textiles-wearing apparel-leather

Mapping from GTAP-EG sectors to model sectors

<i>Energy</i>		
COL	Coal	COL
CRU	Crude oil	CRU
GAS	Natural gas	GAS
OIL	Refined oil products	OIL
ELE	Electricity	ELE
<i>Non-Energy</i>		
EIS	Energy-intensive sectors	CRP, I_S, NFM, NMM, PPP, TRN
Y	Rest of industry	T_T, ATP, AGR, OME, OMN, FPR, LUM, CNS, TWL, OMF, SER, DWE

Table 10: Regional Aggregation

Regions in GTAP-EG

ARG	Argentina	MYS	Malaysia
AUS	Australia	NZL	New Zealand
BRA	Brazil	PHL	Philippines
CAM	Central America and Caribbean	RAP	Rest of Andean Pact
CAN	Canada	RAS	Rest of South Asia
CEA	Central European Associates	REU	Rest of EU
CHL	Chile	RME	Rest of Middle East
CHN	China	RNF	Rest of North Africa
COL	Columbia	ROW	Rest of World
DEU	Germany	RSA	Rest of South Africa
DNK	Denmark	RSM	Rest of South America
EFT	European Free Trade Area	RSS	Rest of South-Saharan Africa
FIN	Finland	SAF	South Africa
FSU	Former Soviet Union	SGP	Singapore
GBR	United Kingdom	SWE	Sweden
HKG	Hong Kong	THA	Thailand
IDN	Indonesia	TUR	Turkey
IND	India	TWN	Taiwan
JPN	Japan	URY	Uruguay
KOR	Republic of Korea	USA	United States of America
LKA	Sri Lanka	VEN	Venezuela
MAR	Morocco	VNM	Vietnam
MEX	Mexico		

Mapping from GTAP-EG regions to model regions from table 1

EUR	Western Europe	GBR, DEU, ITA, NLD, CEA, DNK, EFT, FIN, REU, SWE
JPN	Japan	JPN
USA	United States	USA
CHN	China	CHN, HKG, TWN
FSU	Former Soviet Union	FSU
ROW	Rest of the World	ARG, AUS, BRA, CAM, CAN, CHL, COL, IDN, IND, KOR, LKA, MAR, MEX, MYS, NZL, PHL, RAP, RAS, RME, RNF, ROW, RSA, RSM, RSS, SAF, SGP, THA, TUR, URY, VEN, VNM